

Flavor constraints in a Bosonic Technicolor model

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Flavor constraints in a bosonic Technicolor model are considered. We illustrate different sources for their origin, and emphasize in particular the role played by the vector states present in the Technicolor model. This feature is the essential difference in comparison to an analogous model with two fundamental Higgs scalar doublets.

PACS numbers:

I. INTRODUCTION

Technicolor [1, 2] remains as one of the compelling alternatives to electroweak symmetry breaking extending the simple Higgs sector of the Standard Model (SM). Earliest models based on scaled up QCD run into trouble with precision data [3]. Recent developments suggest that viable models are obtained via introduction of technifermions transforming under higher representations of the gauge group [4, 5]. In this paper we study one of these new models, the so called Next to Minimal Walking Technicolor (NMWT). In NMWT model the Technicolor gauge group is $SU(3)$, and the technicolor matter is constituted by two Dirac flavors transforming under the two-index symmetric, i.e. the sextet representation of the gauge group. Phenomenology of this Technicolor theory has been studied in [6]. Its properties on the lattice have also been investigated [7–11].

The Technicolor interaction only explains the mass patterns in the gauge sector of the Standard Model (SM), and to explain the fermion mass patterns and hierarchies extensions are needed. One possibility is so-called Extended Technicolor (ETC) [12, 13], where one imagines the technifermions and ordinary fermions to be combined under a single representation of a larger gauge group whose breaking then induces effective low-energy four-fermion interaction terms. These, in turn, lead to mass terms for SM fermions as the technifermions condense. However, alternative avenues for addressing the fermion mass generation in Technicolor framework do exist. Instead of assuming the existence of an ETC sector, one may reintroduce scalars with renormalizable Yukawa interactions with ordinary and technicolored matter fields. This is so called bosonic Technicolor [14–18]. To naturalize the scalars, one can supersymmetrize Technicolor [19–21].

Generally, when considering any of the above mentioned extensions of a technicolor model one must pay attention to possibly large contributions to the flavor changing neutral current (FCNC) processes. On the other hand, the fact that already the underlying Technicolor theory contributes to flavor physics has received less attention in the literature [22]. This is so since in Technicolor, the spin-one composite objects and the electroweak gauge bosons with identical quantum numbers will mix providing additional contributions to the flavor observables via the usual box diagrams. So, given a Technicolor model and its bosonic extension, one will be able to constrain both sectors via flavor physics.

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In this paper we consider a bosonic technicolor model obtained by coupling the NMWT model with a SM-like scalar doublet [23]. We do not specify if this is fundamental scalar, possibly a low energy remnant of some supersymmetric scenario, or a composite with the compositeness scale much higher than the energies where the phenomenology of this theory is studied. With a bottom-up model building attitude we simply treat the effective Lagrangian, to be introduced in the next section, as a low energy effective theory and study its associated phenomenology.

We will derive the results required for the analysis of the flavor constraints in this model, but the developed formalism can be applied to other model setups extending beyond basic Technicolor models. In addition, another source of constraints we consider is provided by the precision electroweak data, i.e. the oblique corrections. As we will see, they constrain the spectrum of the bosonic Technicolor in an interesting way, and also mix with the flavor constraints due to the Weinberg sum rules.

As a conclusion, we find that the bosonic NMWT model is viable in light of the present data. However, the constraints do impose severe restrictions on the parameter space of the model. We will determine the results on the masses and mixing patterns of the states. In particular, the compatibility with the precision data inescapably leads to the prediction that at least one of the neutral scalars in the model has to be light, with a mass below 200 GeV. This gives an important phenomenological handle on the model with respect to the LHC data even if the couplings of the lightest scalar are weaker than those of the SM Higgs.

II. BOSONIC NEXT TO MINIMAL WALKING TECHNICOLOR

The low energy effective Lagrangian for Bosonic NMWT has been introduced in detail in [23], but for completeness we recall the basic results here. The effective Lagrangian is, schematically,

$$\mathcal{L} = \mathcal{L}_{\text{TC}}^{\text{eff}}|_{\text{Higgs}=0} + \mathcal{L}_{\text{SM}}|_{\text{Higgs}=0} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}. \quad (1)$$

Here $\mathcal{L}_{\text{TC}}^{\text{eff}}|_{\text{Higgs}=0}$ is almost the same as the effective theory of the NMWT constructed in [6, 26, 32]. The only difference is that $\mathcal{L}_{\text{TC}}^{\text{eff}}|_{\text{Higgs}=0}$ above does not include the *composite* higgs scalar explicitly, i.e. $\mathcal{L}_{\text{TC}}^{\text{eff}}|_{\text{Higgs}=0}$ has only the electroweak gauge bosons and the vector mesons in the sense of the generalized hidden local symmetry [24, 25]. $\mathcal{L}_{\text{SM}}|_{\text{Higgs}=0}$ contains the contribution of the SM degrees of freedom excluding the fundamental Higgs scalar doublet. Finally, $\mathcal{L}_{\text{Higgs}}$ is

$$\mathcal{L}_{\text{Higgs}} = |D_\mu M|^2 + |D_\mu H|^2 + (M, H\text{-mixing term}) - V(M, H), \quad (2)$$

where M is the *composite* scalar field formed by the walking technicolor dynamics and H is another scalar field. As discussed in the introduction we do not specify if H is a fundamental or a composite object. In general M, H are technicolor singlets and have the same EW-charge under $SU(2)_L \times U(1)_Y$ as the SM-Higgs field. Hence, due to the underlying symmetry the H and M fields mix with each other and this is accounted for by the $(M, H\text{-mixing term})$ in Eq.(2). This mixing is resolved via transformation [23]

$$\begin{pmatrix} M \\ H \end{pmatrix} = \begin{pmatrix} A & B \\ -A & B \end{pmatrix} \begin{pmatrix} M_- \\ M_+ \end{pmatrix}, \quad (3)$$

where

$$A = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1 - \frac{c_3}{\alpha} y_Q}}, \quad B = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1 + \frac{c_3}{\alpha} y_Q}}. \quad (4)$$

The parameter c_3 is $\mathcal{O}(1)$ low energy constant, $\alpha = \Lambda/f \sim 4\pi$ is the ratio between the mass scale of the lightest non-Goldstone mode and the Goldstone boson decay constant f . Finally, y_Q is the Yukawa coupling between techniquarks and the scalar doublet H . These parameters originate from the underlying Lagrangian, for details, see [23]. Note that since the mixing of M and H occurs also on the level of the kinetic term, the transformation in (3) is not simply a rotation.

After the kinetic terms in Eq.(2) are diagonalized

$$\mathcal{L}_{\text{Higgs}} = |D_\mu M_+|^2 + |D_\mu M_-|^2 - V(M_\pm), \quad (5)$$

the physical mass eigenstates of the scalar sector are obtained via an additional rotation. We rewrite M_\pm as

$$\begin{pmatrix} G \\ \Pi \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} M_- \\ M_+ \end{pmatrix}, \quad (6)$$

where G indicates the would-be Nambu-Goldstone boson part which is absorbed in the longitudinal mode of the EW-gauge bosons and Π indicates the physical part which will remain in the spectrum. Thus after combining these two transformations, M, H can be represented by G, Π as

$$\begin{pmatrix} M \\ H \end{pmatrix} = \begin{pmatrix} Ac_\beta + Bs_\beta & -As_\beta + Bc_\beta \\ -Ac_\beta + Bs_\beta & As_\beta + Bc_\beta \end{pmatrix} \begin{pmatrix} G \\ \Pi \end{pmatrix}, \quad (7)$$

where we defined $c_\beta \equiv \cos \beta$ and $s_\beta \equiv \sin \beta$.

We parametrize the EW-doublets G, Π as

$$G = \begin{pmatrix} iG^+ \\ \frac{v_{\text{EW}} + h^0 - iG^0}{\sqrt{2}} \end{pmatrix}, \quad \Pi = \begin{pmatrix} i\pi^+ \\ \frac{H^0 - i\pi^0}{\sqrt{2}} \end{pmatrix}, \quad (8)$$

where $M_W^2 = g_{\text{EW}}^2 v_{\text{EW}}^2 / 4$ in which g_{EW} is the physical $SU(2)_L$ gauge coupling.

The final contribution in (1), $\mathcal{L}_{\text{Yukawa}}$, includes the SM Yukawa sector. The Yukawa couplings of the SM quarks are given by

$$\mathcal{L}_{\text{Yukawa},q} = -\tilde{Y}_d^{ij} \tilde{q}_L^i H \tilde{d}_R^j - \tilde{Y}_u^{ij} \tilde{q}_L^i (i\sigma_2 H^*) \tilde{u}_R^j + \text{h.c.}, \quad (9)$$

where we assume that only H has Yukawa coupling with the SM fermions (denoted with a tilde) in the weak gauge eigenbasis. The parameters $\tilde{Y}_{u,d}$ are the Yukawa matrices and have non-diagonal components in general. In terms of the physical mass eigenstates the scalar field H is represented in the unitary gauge as

$$H = (-Ac_\beta + Bs_\beta) \begin{pmatrix} 0 \\ \frac{v_{\text{EW}} + h^0}{\sqrt{2}} \end{pmatrix} + (As_\beta + Bc_\beta) \begin{pmatrix} i\pi^+ \\ \frac{H^0 - i\pi^0}{\sqrt{2}} \end{pmatrix}. \quad (10)$$

Now we divide $\mathcal{L}_{\text{Yukawa},q}$ into three parts as

$$\mathcal{L}_{\text{Yukawa},q} = \mathcal{L}_{\text{Yukawa}}(\text{mass}) + \mathcal{L}_{\text{Yukawa}}(h^0, H^0) + \mathcal{L}_{\text{Yukawa}}(\pi^\pm), \quad (11)$$

where, the first term becomes fermion mass term after changing from the weak basis to the mass eigenbasis for fermions, the second one includes physical neutral higgs and the third one includes physical charged pions. Since only one of the Higgs doublets couples to fermions, the tree level contributions to FCNC interactions are absent in this model, and in our analysis of flavor constraints it suffices to concentrate on the first and third terms of (11). The first term in Eq.(11) is represented as

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}}(\text{mass}) &= -\frac{v_{\text{EW}}(-Ac_\beta + Bs_\beta)}{\sqrt{2}} \left[\tilde{d}_L^i \tilde{Y}_d^{ij} \tilde{d}_R^j - \tilde{u}_L^i \tilde{Y}_u^{ij} \tilde{u}_R^j + \text{h.c.} \right] \\ &= -\frac{v_{\text{EW}}(-Ac_\beta + Bs_\beta)}{\sqrt{2}} \left[\tilde{d}_L^i Y_d^{ii} d_R^i - \tilde{u}_L^i Y_u^{ii} u_R^i + \text{h.c.} \right], \end{aligned} \quad (12)$$

where on the second line the SM fermions without tilde are in the fermion mass eigenbasis and the fermion mass matrices are given by

$$m_{u(d)}^{ii} = \frac{v_{\text{EW}}(-Ac_\beta + Bs_\beta)}{\sqrt{2}} \begin{pmatrix} Y_{u(d)}^{11} & & \\ & Y_{u(d)}^{22} & \\ & & Y_{u(d)}^{33} \end{pmatrix} \equiv \begin{pmatrix} m_{u(d)} & & \\ & m_{c(s)} & \\ & & m_{t(b)} \end{pmatrix}. \quad (13)$$

In the fermion mass eigenbasis, the third term in Eq.(11) is

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}}(\pi^\pm) &= -i\pi^+(As_\beta + Bc_\beta) \left[\bar{u}_L^i Y_d^{jj} d_R^j - \bar{u}_R^i Y_u^{ii} d_L^j \right] V_{ij} + \text{h.c.} \\ &= -i\pi^+ X \left[\bar{u}_L^i m_d^{jj} d_R^j - \bar{u}_R^i m_u^{ii} d_L^j \right] V_{ij} + \text{h.c.}, \end{aligned} \quad (14)$$

where we defined

$$X \equiv \frac{As_\beta + Bc_\beta}{-Ac_\beta + Bs_\beta}. \quad (15)$$

As is evident from (14), the constraints will be most conveniently imposed on the parameter X . This will also hide the parameters of the underlying theory defined briefly below Eq. (4) from the analysis into a single quantity.

III. OBLIQUE CORRECTIONS

The oblique corrections S and T [3] are defined as

$$\begin{aligned} S &= -16\pi\Pi'_{3Y}(0), \\ T &= \frac{4\pi}{s_w^2 c_w^2 M_Z^2} (\Pi_{11}(0) - \Pi_{33}(0)). \end{aligned} \quad (16)$$

To evaluate the corrections we need to estimate the new contributions to the vacuum polarizations

$$\Pi_{3Y}(q^2), \quad \Pi_{ii}(q^2) \quad (i = 1, 2), \quad (17)$$

arising from pion and scalar loop diagrams. The relevant diagrams and Feynman rules following from the expansion of (2) are given in [23], and we will not repeat the calculation in full here. The reader should note, however, that the definition for the mixing angle θ used in [23] differs from our β . The angles are related by

$$\beta = \frac{\pi}{4} - \theta. \quad (18)$$

Taking that into account, we can use the results provided in [23]. Generally, the S parameter is given by

$$S = S_{V,A} + S_S, \quad (19)$$

where $S_{V,A}$ indicates the contribution from the vector mesons V, A and S_S indicates the contribution from the scalar mesons π and the Higgs bosons. Here we will only consider the contribution of the scalar and pseudoscalar degrees of freedom, i.e. S_S . The contribution of the vector mesons is discussed in section V.

The origin of the (S, T) -plane corresponds to the SM with a given value of the mass of the Higgs boson denoted by m_{ref} . We have removed the SM Higgs sector and added new sectors as described in the previous section. Thus the S-parameter is

$$S = S_{\text{SM}}(m_{\text{ref}}) - S_H(m_{\text{ref}}) + S_{\text{new}} = S_{\text{new}} - S_H(m_{\text{ref}}), \quad (20)$$

because $S_{\text{SM}}(m_{\text{ref}}) = 0$ by definition. Similar considerations apply to T , and using these definitions we obtain finite expressions for S and T . We use dimensional regularization in the \overline{MS} scheme, and find that the final result can be expressed in terms of two integrals,

$$\begin{aligned} I_1(m_1, m_2, q) &= \int_0^1 dx \Delta \log \frac{\mu^2}{\Delta}, \\ I_2(m_1, m_2, q) &= \int_0^1 dx m_1^2 \log \frac{\mu^2}{\Delta}, \end{aligned} \quad (21)$$

where $\Delta \equiv \Delta(m_1, m_2, q) = xm_2^2 + (1-x)m_1^2 - x(1-x)q^2$ and μ is an arbitrary mass scale. Furthermore, for notational convenience we define

$$\begin{aligned} C_h &= \frac{1}{v_{\text{EW}}} (f_+ \cos \beta - f_- \sin \beta), \\ C_s &= \frac{1}{v_{\text{EW}}} (f_- \cos \beta + f_+ \sin \beta), \end{aligned} \quad (22)$$

where $f_+ = (f + v)/(2B)$, $f_- = (f - v)/(2A)$. The parameters A and B were defined in (4) and f and v are the vacuum expectation values of the fields M and H , respectively. Similar quantities, f_+ and f_- , referring to the mass eigenbasis of the scalars are related to the electroweak scale v_{EW} by $f_+^2 + f_-^2 = v_{\text{EW}}^2$, so that $C_h^2 + C_s^2 = 1$.

With these preliminary definitions we have

$$\begin{aligned} \Pi_{3Y}(q^2) &= \frac{1}{32\pi^2} [-C_h^2 (I_1(m_\pi, m_{h^0}) - 2I_2(M_Z, m_{H^0}) + I_1(M_Z, m_{H^0})) \\ &\quad - C_s^2 (I_1(m_\pi, m_{H^0}) - 2I_2(M_Z, m_{h^0}) + I_1(M_Z, m_{h^0})) \\ &\quad + I_1(m_\pi, m_\pi) + I_1(M_Z, m_{\text{ref}}) - 2I_2(M_Z, m_{\text{ref}})], \end{aligned} \quad (23)$$

where we have dropped all q -independent contributions since we need only the q^2 -derivative of this quantity. Note that here and later m_{h^0} and m_{H^0} are the masses of the physical scalars h^0 and H^0 . The contributions to the correlators needed for the T -parameter are obtained similarly, and the result is

$$\begin{aligned} \Pi_{11}(q^2) &= \frac{1}{32\pi^2} [C_h^2 (I_1(M_W, m_{H^0}) + I_1(m_\pi, m_{h^0}) - 2I_2(M_W, m_{H^0})) \\ &\quad + C_s^2 (I_1(M_W, m_{h^0}) + I_1(m_\pi, m_{H^0}) - 2I_2(M_W, m_{h^0})) \\ &\quad + I_1(m_\pi, m_\pi) - I_1(M_W, m_{\text{ref}}) + 2I_2(M_W, m_{\text{ref}}) \\ &\quad + \frac{3}{2}m_\pi^2 - \frac{1}{2}m_{\text{ref}}^2 + \frac{1}{2}m_{h^0}^2 + \frac{1}{2}m_{H^0}^2 - \frac{1}{3}q^2], \end{aligned} \quad (24)$$

and

$$\Pi_{33}(q^2) = \Pi_{11}(q^2) \quad \text{with} \quad M_W \rightarrow M_Z. \quad (25)$$

The results of a numerical computation and the corresponding constraints due to the data on S and T will be presented in Sec. V.

IV. FLAVOR CONSTRAINTS

In [22], the contributions to the flavor observables from the vector mesons for the NMWT model have been estimated. The analysis of [22] was carried out in terms of the weak eigenstates of the vector mesons. In the present paper, we reconsider the analysis of [22] in the mass basis and extend it to the

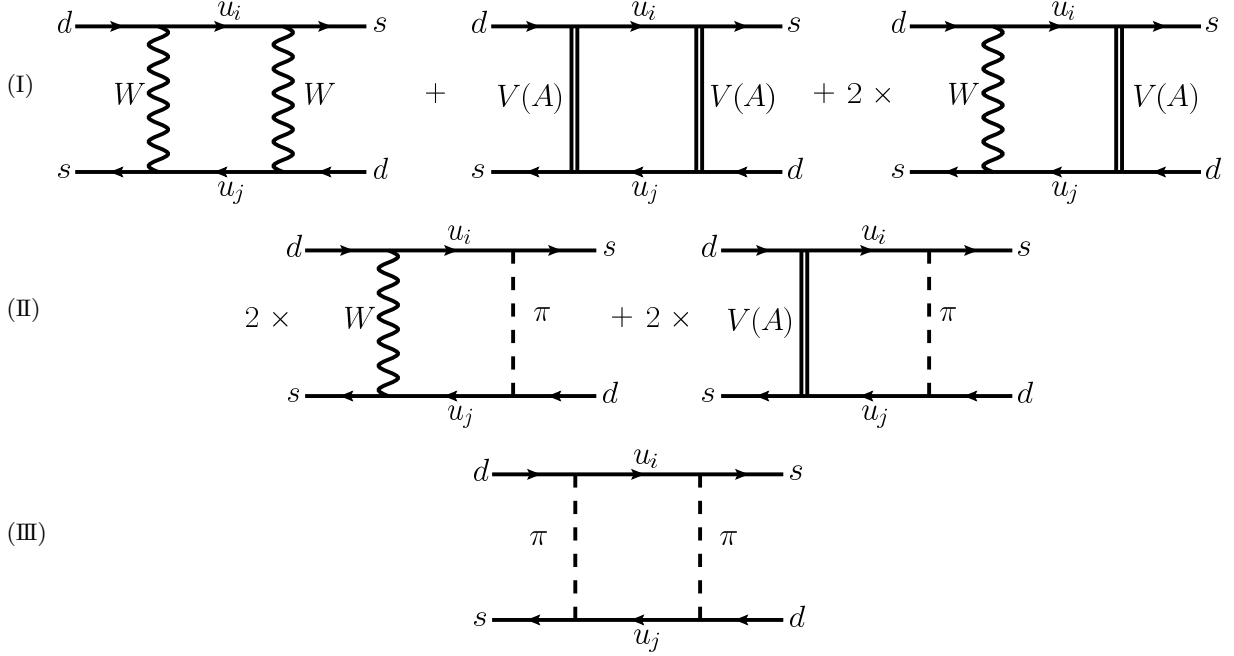


FIG. 1: Box diagrams for $\Delta S = 2$ scattering process in the unitary gauge. To obtain $\Delta B = 2$ process, we should simply rename s with b and d with q ($q = d, s$) in the various diagrams.

case of bosonic NMWT. For this purpose, we should first determine the eigenvalues and eigenstates for the mass matrix of the vector bosons. In this paper, we take into consideration $\Delta F = 2$ constraints, so we concentrate on the charged vector bosons sector. Here F labels the flavor relevant for each of the processes we consider. Thus we can represent the charged current coupling to the fermions as

$$\begin{aligned} & \frac{g}{\sqrt{2}} \left[V_{ij}^* \cdot \tilde{W}_\mu^- \bar{d}_L^j \gamma^\mu u_L^i + \text{h.c.} \right] \\ &= \frac{g}{\sqrt{2}} \left[V_{ij}^* \cdot (x_W W_\mu^- + x_V V_\mu^- + x_A A_\mu^-) \bar{d}_L^j \gamma^\mu u_L^i + \text{h.c.} \right], \end{aligned} \quad (26)$$

where the tilde notation refers to the weak basis and the fields and parameters without tilde refer to the mass basis. The CKM matrix is denoted by V . The notations and complete expressions for the mass eigenvalues and eigenvectors of vector mesons are given in Appendix A.

To analyse the flavor changing neutral current interactions in the bosonic NMWT model, we compute the Box diagrams contributing to the $\Delta F = 2$ processes. The required diagrams are shown in Fig. 1. For these box diagrams we obtain results

$$\text{(I)} = \frac{-ig_{\text{EW}}^4}{(4\pi)^2 \cdot 4M_W^2} \sum_{i,j} \lambda_i \lambda_j \mathcal{F}^{(1)}(m_i, m_j, M_W, M_V, M_A) \times \mathcal{O}_{\Delta F=2}, \quad (27)$$

$$\text{(II)} = \frac{-ig_{\text{EW}}^4 X^2}{(4\pi)^2 \cdot 4M_W^2} \sum_{i,j} \lambda_i \lambda_j \mathcal{F}^{(2)}(m_i, m_j, M_W, \{M\}) \times \mathcal{O}_{\Delta F=2}, \quad (28)$$

$$\text{(III)} = \frac{-ig_{\text{EW}}^4 X^4}{(4\pi)^2 \cdot 4M_W^2} \sum_{i,j} \lambda_i \lambda_j \mathcal{F}^{(3)}(m_i, m_j, M_W, \{M\}) \times \mathcal{O}_{\Delta F=2}, \quad (29)$$

where $\{M\}$ stands for the collection $\{M_V, M_A, M_\pi\}$, the operator $\mathcal{O}_{\Delta F=2}$ is

$$\mathcal{O}_{\Delta F=2} = \begin{cases} (\bar{s}_L \gamma^\mu d_L)(\bar{s}_L \gamma_\mu d_L) & \text{for } F = S, \\ (\bar{b}_L \gamma^\mu q_L)(\bar{b}_L \gamma_\mu q_L) & \text{for } F = B_q, (q = d, s). \end{cases} \quad (30)$$

and λ_i is given in terms of the CKM matrix elements by

$$\lambda_i = \begin{cases} V_{id} V_{is}^* & \text{for } F = S, \\ V_{iq} V_{ib}^* & \text{for } F = B_q, (q = d, s). \end{cases} \quad (31)$$

The functions $\mathcal{F}^{(1,2,3)}$ are given explicitly in Appendix B.

We have used the physical $SU(2)_L$ gauge coupling g_{EW} , which is related to the bare gauge coupling g as

$$\frac{1}{g_{EW}^2} = \frac{1}{g^2} \left[1 + \frac{1 + (1 - \chi)^2}{2} \epsilon^2 \right] = \frac{1}{g^2} \left[1 + \frac{1 + (1 - \chi)^2}{2} \bar{\epsilon}^2 \right], \quad (32)$$

where χ is a numerical factor introduced as in [26], and whose value will be determined in Sec. V. This result is easily obtained by equating the well known Fermi coupling $G_F \sim g_{EW}^2/M_W^2$ with the similar quantity g^2/\tilde{M}_W^2 which would be identified as the Fermi coupling in case this theory was applied as the Standard Model; see appendix B for the expressions relating the parameters of the gauge and mass eigenbases. In the first equality we have used the definition $\epsilon = g/\tilde{g}$, where \tilde{g} is the self coupling of the vector mesons, and expanded up to the order $\mathcal{O}(\epsilon^4)$. The second equality is obtained with the definition $\bar{\epsilon} = g_{EW}/\tilde{g}$, and noting that $\bar{\epsilon}^2 = \epsilon^2$ up to contributions of order $\mathcal{O}(\bar{\epsilon}^4)$. For the results we present in this paper the quantity $\bar{\epsilon}$ is more convenient than ϵ since g_{EW} is the relevant coupling for the physical states.

In Appendix B we have collected the expressions relating the gauge and mass eigenbases up to $\mathcal{O}(\epsilon^5)$, but in the applications here, we will generally neglect $\mathcal{O}(\bar{\epsilon}^3)$ and higher terms in all expressions.

In total, then, the $\Delta F = 2$ FCNC process is described by an effective Lagrangian given by

$$\mathcal{L}(\Delta F = 2) = -\frac{G_F^2 M_W^2}{4\pi^2} \cdot \mathcal{F}_0(m_i, m_j, M_W, M_V, M_A, M_\pi) \cdot \mathcal{O}_{\Delta F=2}, \quad (33)$$

where

$$\begin{aligned} \mathcal{F}_0(m_i, m_j, M_W, \{M\}) &= \sum_{i,j} \lambda_i \lambda_j \left[\mathcal{F}^{(1)} + X^2 \mathcal{F}^{(2)} + X^4 \mathcal{F}^{(3)} \right] \\ &= \mathcal{F}_0^{\text{SM}}(m_i, m_j, M_W) + \Delta \mathcal{F}_0(m_i, m_j, M_W, \{M\}). \end{aligned} \quad (34)$$

Here X is defined in (15), and we have decomposed the result into a SM contribution, $\mathcal{F}_0^{\text{SM}} \equiv \sum \lambda_i \lambda_j \mathcal{F}_1(m_i, m_j, M_W)$, and a contribution arising solely from the new physics,

$$\Delta \mathcal{F}_0 = \Delta \mathcal{F}_0^s(m_i, m_j, M_W, M_\pi) + \bar{\epsilon}^2 \Delta \mathcal{F}_0^{\text{vm}}(m_i, m_j, M_W, M_V, M_A, M_\pi). \quad (35)$$

Furthermore, here $\Delta \mathcal{F}_0^s$ is the contribution solely from π^\pm exchange and $\bar{\epsilon}^2 \Delta \mathcal{F}_0^{\text{vm}}$ is the contribution from diagrams including vector mesons together with the charged pions.

When we estimate the flavor observables in our setting, we take into account the unitarity of the CKM matrix by imposing $m_u \rightarrow 0$ [27]. For example in the case of $F = S$,

$$\mathcal{F}_0 = \eta_1 \cdot \lambda_c^2 \bar{\mathcal{F}}_0(m_c, M_W, \{M\}) + \eta_2 \cdot \lambda_t^2 \bar{\mathcal{F}}_0(m_t, M_W, \{M\}) + \eta_3 \cdot 2\lambda_c \lambda_t \bar{\mathcal{F}}_0(m_c, m_t, M_W, \{M\}), \quad (36)$$

where $\eta_{1,2,3}$ encode the QCD corrections for each $\bar{\mathcal{F}}_0$, and which are given by $\eta_1 = 1.44$, $\eta_2 = 0.57$, $\eta_3 = 0.47$ [28]. We note that, in the case of $F = B$ we need only $\eta_2 = \eta_B = 0.55$ [28] because the top quark dominates the contributions to the box diagrams for $\Delta B = 2$ processes. The function $\bar{\mathcal{F}}_0$ is given by

$$\bar{\mathcal{F}}_0(m_i, M_W, \{M\}) = \lim_{m_u \rightarrow 0} \bar{\mathcal{F}}_0(m_u, m_i, M_W, \{M\}), \quad (37)$$

parameter	68% probability	95% probability
C_{ϵ_K}	1.09 ± 0.13	$[0.86, 1.39]$
$C_{\Delta M_K}$	1.17 ± 0.42	$[0.61, 2.43]$
C_{B_d}	0.97 ± 0.14	$[0.72, 1.30]$
C_{B_s}	0.96 ± 0.10	$[0.79, 1.18]$

TABLE I: Constraints for C_{ϵ_K} , $C_{\Delta M_K}$ and C_{B_q} [31].

and we use the charm quark mass $m_c = 1.3 \text{ GeV}$ [29], and the top quark mass $m_t = 163.4 \text{ GeV}$ [29].

In accordance with [30], we define $C_{\epsilon_K, \Delta M_K, B_q}$ as

$$C_{\epsilon_K} \equiv \frac{\text{Im} [\langle K^0 | \mathcal{H}_{\text{eff}}^{\text{full}} | K^{\bar{0}} \rangle]}{\text{Im} [\langle K^0 | \mathcal{H}_{\text{eff}}^{\text{SM}} | K^{\bar{0}} \rangle]} = 1 + \frac{\text{Im} [\Delta \mathcal{F}_0(F = S)]}{\text{Im} [\mathcal{F}_0^{\text{SM}}(F = S)]}, \quad (38)$$

$$C_{\Delta M_K} \equiv \frac{\text{Re} [\langle K^0 | \mathcal{H}_{\text{eff}}^{\text{full}} | K^{\bar{0}} \rangle]}{\text{Re} [\langle K^0 | \mathcal{H}_{\text{eff}}^{\text{SM}} | K^{\bar{0}} \rangle]} = 1 + \frac{\text{Re} [\Delta \mathcal{F}_0(F = S)]}{\text{Re} [\mathcal{F}_0^{\text{SM}}(F = S)]}, \quad (39)$$

$$C_{B_q} \equiv \left| \frac{\langle B_q^0 | \mathcal{H}_{\text{eff}}^{\text{full}} | \bar{B}_q^0 \rangle}{\langle B_q^0 | \mathcal{H}_{\text{eff}}^{\text{SM}} | \bar{B}_q^0 \rangle} \right| = \left| 1 + \frac{\Delta \mathcal{F}_0(F = B)}{\mathcal{F}_0^{\text{SM}}(F = B)} \right|, \quad (40)$$

where $\mathcal{F}_0^{\text{SM}}(F = S, B)$ are the SM contributions, $\mathcal{F}_0^{\text{SM}}$ defined earlier, with λ_i in Eq.(31) chosen in correspondence with the process indicated in parentheses by $F = S$ or $F = B$.

The UT-fit collaboration provides constraints on each C parameter defined above; these are collected in Table I. Finally, the CKM matrix is given by [29]

$$V_{\text{CKM}} = \begin{pmatrix} 0.97427 & 0.22535 & 0.00377e^{i(-70.0^\circ)} \\ -0.22525e^{i(-0.0358^\circ)} & 0.97345 & 0.04082 \\ 0.00869e^{i(-23.3^\circ)} & -0.04007e^{i(-1.138^\circ)} & 0.99916 \end{pmatrix}. \quad (41)$$

V. NUMERICAL RESULTS FOR BOSONIC NMWT

Let us start with the oblique corrections. As already mentioned, in our case, the S parameter is given by

$$S = S_{V,A} + S_S, \quad (42)$$

where $S_{V,A}$ indicates the contribution from the vector mesons V, A and S_S indicates the contribution from the scalar mesons π and the Higgs bosons. On the other hand, $S_{V,A}$ is related to the 0th Weinberg sum rule, because this sum rule is derived from vector-axial current correlation functions with vector meson saturation, and this does not have any contribution from physical scalar mesons. The 0th Weinberg sum rule is represented as [32]

$$S_{V,A} = 4\pi \left[\frac{F_V^2}{\tilde{M}_V^2} - \frac{F_A^2}{\tilde{M}_A^2} \right] = \frac{8\pi}{\tilde{g}^2} [1 - (1 - \chi)^2] + \mathcal{O}(\tilde{\epsilon}^2). \quad (43)$$

The parameter χ is related to the parameters of the underlying theory, but is in principle free to take any value and hence $S_{V,A}$ can be vanishing or even negative. For our analysis, we will take $S_{V,A} = 0$, i.e. $\chi = 1$. The calculation of S_S was detailed in Sec. III, and here we turn directly to the numerical results.

We demand that the values of S and T are within the 90% confidence limit as obtained by the PDG [33]. The resulting constraints for the pion mass as well as the masses of the scalars are shown in figure 2. As already noted in [23], the electroweak precision constraints favor a scenario of one light and one heavy scalar particle, whereas the pion mass is not very strictly limited by the electroweak precision data.

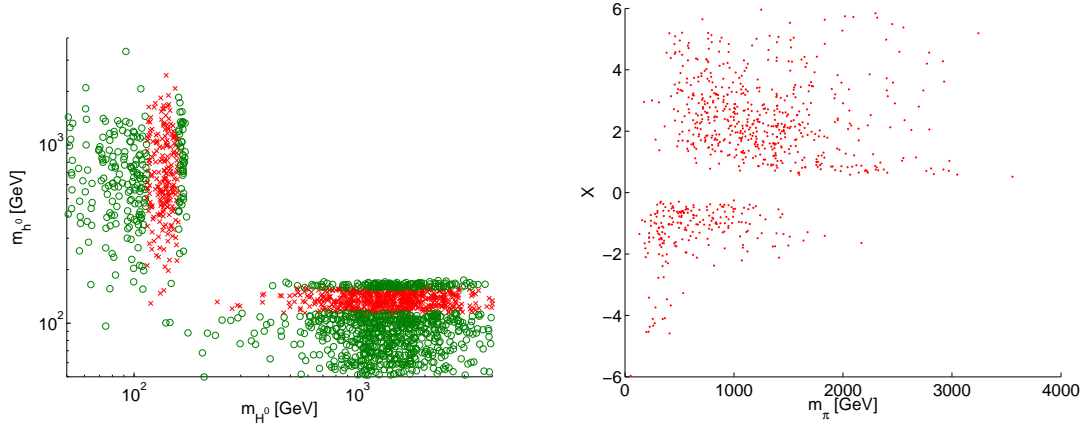


FIG. 2: Left panel: The masses of the scalars h^0 and H^0 after the constraints from S and T have been applied. The points marked by green circles are ruled out by direct detection limits from LEP and Tevatron, assuming SM-Higgs like branching ratios. The red crosses are allowed. Right panel: the mass of the pion and the variable X as defined in equation (15), after the electroweak precision constraints and the direct detection limits have been applied.

Then we turn to the analysis of the flavor constraints. As detailed in Sec. IV, we have six parameters at our disposal. These are $X, M_V, M_A, M_\pi, \tilde{g}$ and χ . To reduce the number of free parameters further we apply Weinberg sum rules. In addition to the 0th sum rule given above, the 1st Weinberg sum rule is given by [32]

$$v_{EW}^2 = F_V^2 - F_A^2, \quad (44)$$

where v_{EW} satisfies $v_{EW}^2 = 4\tilde{M}_W^2/\tilde{g}_{EW}^2 = 4M_W^2/g_{EW}^2$. So the 1st Weinberg sum rule becomes

$$M_W^2 = \frac{\epsilon^2}{2} [M_V^2 - (1 - \chi)^2 M_A^2], \quad (45)$$

where we neglect $\mathcal{O}(\epsilon^4)$ contributions.

Thus, with the two Weinberg sum rules we reduce the parameters as follows: With eqs.(43) and given $S_{V,A} = 0$, we have $\chi = 1$. Then via (45), we express M_V in terms of the other parameters and this leaves \tilde{g}, M_A, M_π and X as free parameters. For our analysis, we choose certain benchmark values for \tilde{g} and M_A and then scan the (m_π, X) -plane. For each set of parameters we evaluate $\Delta\mathcal{F}$ as defined in (35), and then use the constraints on (38)-(40) to see if the corresponding point in the parameter space is viable.

Results of this procedure for $\chi = 1$, $\tilde{g} = 1, 5$ and $M_A = 400$ GeV or 600 GeV are shown in Fig.3. We checked that we cannot find any significant difference from the present results if the value of $S_{V,A}$ is varied from zero to $1/\pi$, where $S = 1/\pi$ corresponds to the perturbative naive value of S in the

NMWT model. To comment about a difference in the flavor constraints arising from varying $S = 1/\pi$ to $S = 0$ we note the following: The lower bound constraint in the figures coming from C_{ϵ_K} is slightly larger with $S = 0$ than with $S = 1/\pi$. On the other hand, the upper constraint coming from C_{B_s} is slightly smaller with $S = 0$ than with $S = 1/\pi$. These differences are only on the level of $\mathcal{O}(10^{-2}\%)$ for a fixed value of m_π , independently of the values of \tilde{g}, M_A .

However, the constraints on the S and T from precision data are more stringent: allowing for $S_{V,A} \simeq 0.1$ in addition to the scalar contribution would require the presence of a light scalar with mass below 100 GeV which would be difficult to reconcile with phenomenologically, even if its couplings were different from a Standard Model Higgs. In general, the larger the value for $S_{V,A}$ is assumed, the smaller the mass of the lighter one of the scalar boson states needs to be. For example, if we take $S_{V,A} = 0.05$, the parameter space points where the lightest scalar mass is of the order of 170 GeV are ruled out and we are left with points where the mass is of the order of 130 GeV or less. Thus, depending on the value of $S_{V,A}$, the parameter space may be highly restricted by the combination of electroweak precision observables (favoring the existence of a light scalar) and SM Higgs searches. A similar tension on the Higgs mass due to precision measurements and direct observation constraints is also present in the SM itself, as well as in typical supersymmetric extensions of the SM.

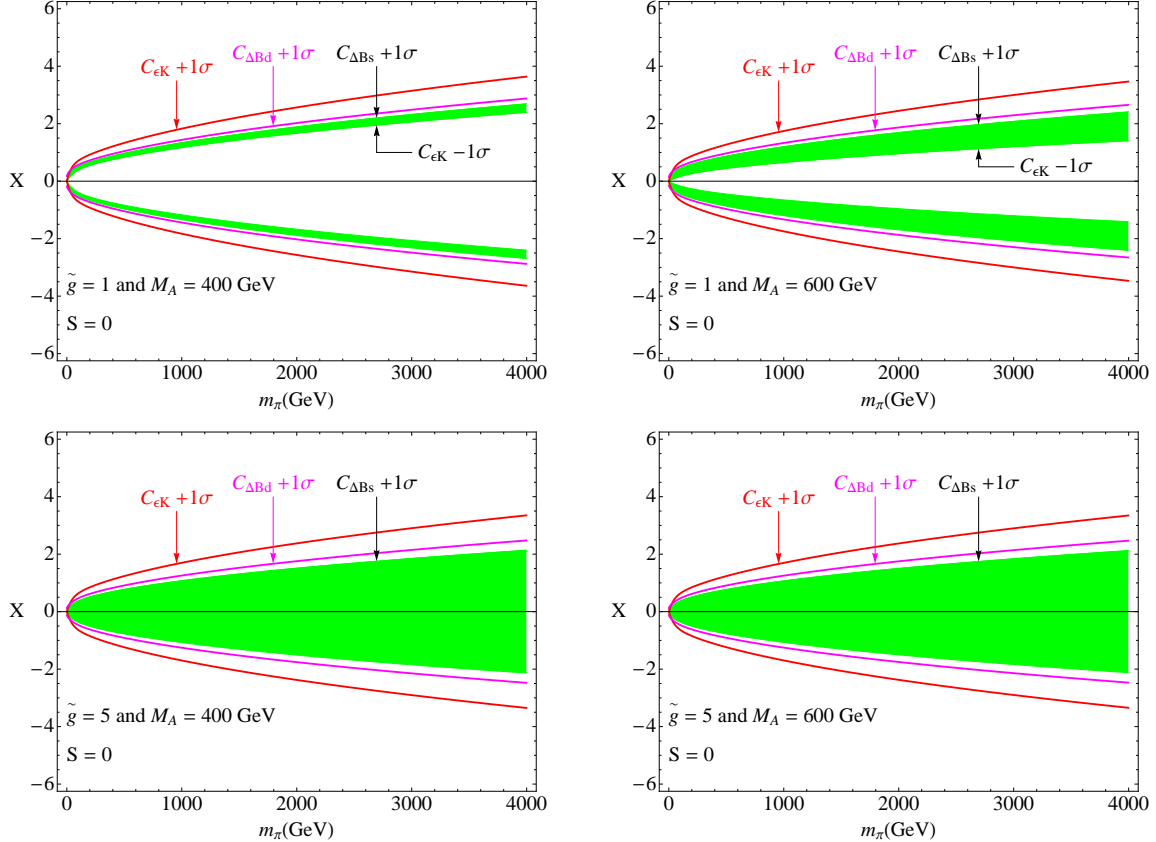


FIG. 3: The UT-fit constraints coming from $C_{\epsilon_K, \Delta M_{K, B_q}}$ (68% probability) on (M_π, X) -plane for $(\tilde{g}, M_A) = (1, 400 \text{ GeV})$ [top-left], $(1, 600 \text{ GeV})$ [top-right], $(5, 400 \text{ GeV})$ [bottom-left] and $(5, 600 \text{ GeV})$ [bottom-right] with $S_{V,A} = 0$. The shaded region is allowed region. We show $C_{\epsilon_K, \Delta M_{K, B_q}}$ (68% probability) constraints in all panels. In the bottom panels, the lower constraints coming from C_{ϵ_K} disappears.

As one can see from Fig. 3, the most severe upper bound comes from C_{B_s} and the lower 68% C.L. constraint comes from C_{ϵ_K} in table I. A lower constraint coming from C_{ϵ_K} disappears if $\tilde{g} \geq 2$ or

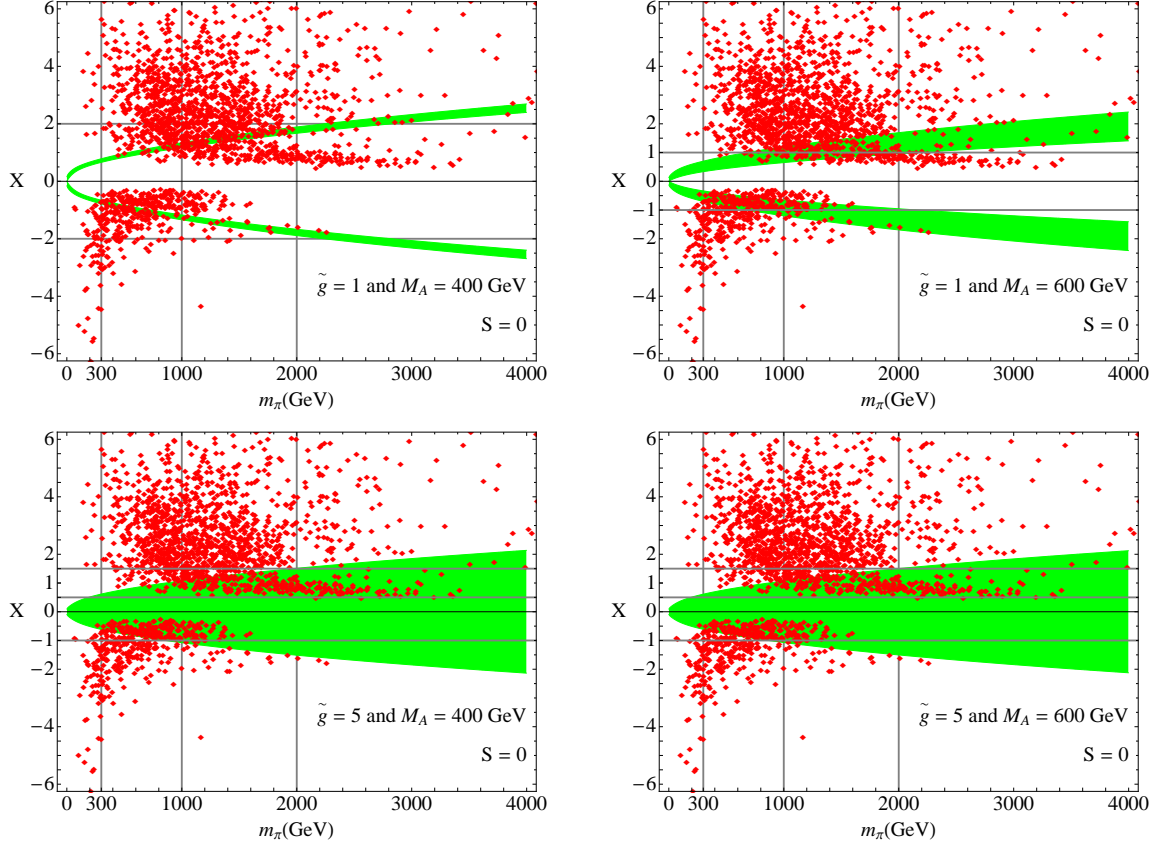


FIG. 4: Constraints from flavor and electroweak oblique corrections on (M_π, X) -plane for $(\tilde{g}, M_A) = (1, 400 \text{ GeV})$ [top-left], $(1, 600 \text{ GeV})$ [top-right], $(5, 400 \text{ GeV})$ [bottom-left] and $(5, 600 \text{ GeV})$ [bottom-right] with $S_{V,A} = 0$. The red diamonds correspond to the red crosses in the right panel in Fig.2. The shaded regions correspond to the shaded region in Fig.3.

$M_A \geq 650 \text{ GeV}$. So, in this case it is enough to take into consideration the flavor constraint coming solely from the upper C_{B_s} constraint. In Fig.4, we show the flavor constraints (shaded region) in Fig.3 with the constraints (red diamonds) from the electroweak oblique corrections for scalar sector in Fig.2. Therefore our analysis seems to indicate that electroweak oblique corrections and flavor constraints taken together prefer

$$\begin{cases} -1 < X < 0 & \text{and } 300 \text{ GeV} < m_\pi < 1.6 \text{ TeV} & \text{for any } \tilde{g} \text{ and } M_A, \\ 1 < X < 2 & \text{and } 800 \text{ GeV} < m_\pi < 2 \text{ TeV} & \text{for } \tilde{g} \leq 2 \text{ and } M_A \leq 650 \text{ GeV}, \\ 0.5 < X < 1.5 & \text{and } 800 \text{ GeV} < m_\pi & \text{for } \tilde{g} \geq 2 \text{ or } M_A \geq 650 \text{ GeV}. \end{cases} \quad (46)$$

In any of these cases, we see that it is possible that $m_\pi > M_A$ in the bosonic NMWT model. This ordering in the spectrum would be different from the case of QCD-like theories.

VI. CONCLUSIONS AND OUTLOOK

In this paper we have revisited the bosonic next-to-minimal Walking Technicolor (NMWT) model introduced in [23]. We have extended the viability analysis of the model by considering the flavor constraints. The model, and the analysis carried out here, illustrate an important generic feature of flavor constraints in the Technicolor context: while there are the contributions due to the extension of the basic Technicolor theory, there are also intrinsic contributions arising from the Technicolor theory itself. These latter contributions originate from the vector mesons which mix with the electroweak gauge bosons. The former contributions were evaluated in this paper for bosonic Technicolor, but clearly similar analysis should be carried out for models where fermion mass generation is due to extended Technicolor (ETC) interactions.

As a result of our analysis we conclude that bosonic NMWT remains as a viable low energy model able to provide for masses of the SM matter fields. Of course the hierarchical mass patterns remain unexplained in the sense that they are merely parametrized in terms of Yukawa couplings similarly as e.g. in (MS)SM. Our results show that imposing the flavor constraints together with the constraints from oblique corrections provides bounds on the mixing patterns of the scalar states and masses of the physical pions.

Appendix A: Weak eigenstates

In the NMWT model, mass term of the charged vector mesons is given by [32]

$$\mathcal{L}_{CS} = (\tilde{W}_\mu^- \tilde{V}_\mu^- \tilde{A}_\mu^-) \cdot \tilde{\mathcal{M}}_{CS}^2 \cdot (\tilde{W}^{+\mu} \tilde{V}^{+\mu} \tilde{A}^{+\mu})^T. \quad (\text{A1})$$

Here, the tilde notation for the charged vector states indicate the fields in terms of the weak basis and

$$\tilde{\mathcal{M}}_{CS}^2 = \begin{pmatrix} \tilde{M}_W^2 & -\frac{g}{\sqrt{2}\tilde{g}} \cdot \tilde{M}_V^2 & -\frac{g}{\sqrt{2}\tilde{g}} \cdot \tilde{M}_A^2 \cdot (1-\chi) \\ -\frac{g}{\sqrt{2}\tilde{g}} \cdot \tilde{M}_V^2 & \tilde{M}_V^2 & 0 \\ -\frac{g}{\sqrt{2}\tilde{g}} \cdot \tilde{M}_A^2 \cdot (1-\chi) & 0 & \tilde{M}_A^2 \end{pmatrix}, \quad (\text{A2})$$

where notations are according to [32] except for $\tilde{M}_{W,V,A}$ which are given by

$$\tilde{M}_W^2 = \frac{g^2}{\tilde{g}^2} \left[\left(\frac{1}{2} + \omega \right) \tilde{M}_V^2 - \frac{(1-\chi)^2}{2} \tilde{M}_A^2 \right], \quad \tilde{M}_V^2 = \frac{\tilde{g}^2}{2} F_V^2, \quad \tilde{M}_A^2 = \frac{\tilde{g}^2}{2(1-\chi)^2} F_A^2. \quad (\text{A3})$$

The parameters χ and ω are related to the couplings in the underlying Lagrangian and like for $F_{V(A)}$, the (axial)vector-decay constant, their definitions are same as in [32]. The mass matrix $\tilde{\mathcal{M}}_{CS}^2$ is diagonalized by the orthogonal transformation and as a result we obtain the eigenvalues $M_{W,V,A}^2$ which are given by ($\epsilon \equiv g/\tilde{g}$)

$$M_W^2 = \tilde{M}_W^2 \left[1 - \frac{1 + (1-\chi)^2}{2} \cdot \epsilon^2 \right], \quad (\text{A4})$$

$$M_{V^\pm}^2 = \tilde{M}_V^2 \left[1 + \frac{\tilde{M}_W^2 + \tilde{M}_V^2}{2\tilde{M}_V^2} \cdot \epsilon^2 - \frac{(1-\chi)^2 \tilde{M}_A^2}{4(\tilde{M}_A^2 - \tilde{M}_V^2)} \cdot \epsilon^4 \right], \quad (\text{A5})$$

$$M_{A^\pm}^2 = \tilde{M}_A^2 \left[1 + (1-\chi)^2 \frac{\tilde{M}_W^2 + \tilde{M}_A^2}{2\tilde{M}_A^2} \cdot \epsilon^2 - \frac{(1-\chi)^2 \tilde{M}_V^2}{4(\tilde{M}_V^2 - \tilde{M}_A^2)} \cdot \epsilon^4 \right], \quad (\text{A6})$$

where we neglect $\mathcal{O}(\epsilon^5)$ -terms. Moreover, we present the mass eigenvectors as

$$\vec{v}_W = \begin{pmatrix} x_W \\ y_W \\ z_W \end{pmatrix}, \quad \vec{v}_V = \begin{pmatrix} x_V \\ y_V \\ z_V \end{pmatrix}, \quad \vec{v}_A = \begin{pmatrix} x_A \\ y_A \\ z_A \end{pmatrix}, \quad (\text{A7})$$

in which each x, y, z is given by

$$x_W = 1 - \frac{1}{2} \cdot \frac{1 + (1 - \chi)^2}{2} \epsilon^2 + \frac{3}{8} \cdot \frac{[1 + (1 - \chi)^2]^2}{4} \epsilon^4 - \frac{1}{2} \left[\frac{\tilde{M}_W^2}{\tilde{M}_V^2} + (1 - \chi)^2 \frac{\tilde{M}_W^2}{\tilde{M}_A^2} \right] \epsilon^2, \quad (\text{A8})$$

$$y_W = \frac{\epsilon}{\sqrt{2}} \left[1 + \frac{\tilde{M}_W^2}{\tilde{M}_V^2} - \frac{1 + (1 - \chi)^2}{4} \epsilon^2 \right], \quad (\text{A9})$$

$$z_W = \frac{(1 - \chi)\epsilon}{\sqrt{2}} \left[1 + \frac{\tilde{M}_W^2}{\tilde{M}_A^2} - \frac{1 + (1 - \chi)^2}{4} \epsilon^2 \right], \quad (\text{A10})$$

$$x_V = -\frac{\epsilon}{\sqrt{2}} \left[1 + \frac{\tilde{M}_W^2}{\tilde{M}_V^2} - \frac{1}{2} \left(\frac{1}{2} + \frac{(1 - \chi)^2 \tilde{M}_A^2}{\tilde{M}_A^2 - \tilde{M}_V^2} \right) \epsilon^2 \right], \quad (\text{A11})$$

$$y_V = 1 - \frac{\epsilon^2}{4} \left[1 + 2 \frac{\tilde{M}_W^2}{\tilde{M}_V^2} - \frac{1}{2} \left(\frac{3 + 4(1 - \chi)^2}{4} - \frac{(1 - \chi)^2 \tilde{M}_V^4}{(\tilde{M}_A^2 - \tilde{M}_V^2)^2} \right) \epsilon^2 \right], \quad (\text{A12})$$

$$z_V = \frac{\epsilon^2 \cdot (1 - \chi) \tilde{M}_A^2}{2(\tilde{M}_V^2 - \tilde{M}_A^2)} \left[1 + \frac{\tilde{M}_W^2}{\tilde{M}_V^2} - \frac{1}{2} \left(\frac{1}{2} - \frac{\tilde{M}_V^2 - (1 - \chi)^2 \tilde{M}_A^2}{\tilde{M}_A^2 - \tilde{M}_V^2} \right) \epsilon^2 \right], \quad (\text{A13})$$

$$x_A = -\frac{(1 - \chi)\epsilon}{\sqrt{2}} \left[1 + \frac{\tilde{M}_W^2}{\tilde{M}_A^2} - \frac{1}{2} \left(\frac{(1 - \chi)^2}{2} + \frac{\tilde{M}_V^2}{\tilde{M}_V^2 - \tilde{M}_A^2} \right) \epsilon^2 \right], \quad (\text{A14})$$

$$y_A = \frac{\epsilon^2 \cdot (1 - \chi) \tilde{M}_V^2}{2(\tilde{M}_A^2 - \tilde{M}_V^2)} \left[1 + \frac{\tilde{M}_W^2}{\tilde{M}_A^2} - \frac{1}{2} \left(\frac{(1 - \chi)^2}{2} + \frac{\tilde{M}_V^2 - (1 - \chi)^2 \tilde{M}_A^2}{\tilde{M}_V^2 - \tilde{M}_A^2} \right) \epsilon^2 \right], \quad (\text{A15})$$

$$z_A = 1 - \frac{(1 - \chi)^2 \epsilon^2}{4} \left[1 + 2 \frac{\tilde{M}_W^2}{\tilde{M}_A^2} - \frac{1}{2} \left(\frac{4 + 3(1 - \chi)^2}{4} - \frac{\tilde{M}_A^4}{(\tilde{M}_V^2 - \tilde{M}_A^2)^2} \right) \epsilon^2 \right], \quad (\text{A16})$$

where we neglect $\mathcal{O}(\epsilon^5)$ again.

In the text we keep terms only up to $\mathcal{O}(\epsilon^3)$ for each estimation.

Appendix B: The box diagrams

The functions arising from the evaluation of the box diagrams are defined as

$$\mathcal{F}^{(1)}(m_i, m_j, M_W, M_V, M_A) = \bar{\mathcal{F}}_1(m_i, m_j, M_W) - \bar{\epsilon}^2 \left[\begin{aligned} & 2 \left(\frac{M_W^2}{M_V^2} + (1 - \chi)^2 \frac{M_W^2}{M_A^2} \right) \bar{\mathcal{F}}_1(m_i, m_j, M_W) \\ & + \frac{(M_V^2 + M_W^2)^2}{M_V^4} \bar{\mathcal{F}}_1(m_i, m_j, M_W, M_V) \\ & + (1 - \chi)^2 \frac{(M_A^2 + M_W^2)^2}{M_A^4} \bar{\mathcal{F}}_1(m_i, m_j, M_W, M_A) \end{aligned} \right], \quad (\text{B1})$$

$$\mathcal{F}^{(2)}(m_i, m_j, M_W, \{M\}) = 2\bar{\mathcal{F}}_2(m_i, m_j, M_W, M_\pi) - \bar{\epsilon}^2 \left[\begin{aligned} & 2 \left(\frac{M_W^2}{M_V^2} + (1 - \chi)^2 \frac{M_W^2}{M_A^2} \right) \bar{\mathcal{F}}_2(m_i, m_j, M_W, M_\pi) \\ & - \frac{(M_V^2 + M_W^2)^2}{M_V^4} \bar{\mathcal{F}}_2(m_i, m_j, M_V, M_\pi) \\ & - (1 - \chi)^2 \frac{(M_A^2 + M_W^2)^2}{M_A^4} \bar{\mathcal{F}}_2(m_i, m_j, M_A, M_\pi) \end{aligned} \right], \quad (\text{B2})$$

$$\mathcal{F}^{(3)}(m_i, m_j, M_W, M_\pi) = \frac{m_i^2 m_j^2}{M_W^2} A_2(m_i, m_j, M_\pi), \quad (\text{B3})$$

and

$$\begin{aligned} \frac{1}{M_W^2} \bar{\mathcal{F}}_1(m_1, m_2, M_1, M_2) &= \left(1 + \frac{m_1^2 m_2^2}{4M_1^2 M_2^2} \right) A_2(m_1, m_2, M_1, M_2) \\ &\quad + \frac{m_1^2 m_2^2 (M_1^2 + M_2^2)}{M_1^2 M_2^2} A_0(m_1, m_2, M_1, M_2), \end{aligned} \quad (\text{B4})$$

$$\frac{1}{M_W^2} \bar{\mathcal{F}}_1(m_1, m_2, M) = \left(1 + \frac{m_1^2 m_2^2}{4M^4} \right) A_2(m_1, m_2, M) + \frac{2m_1^2 m_2^2}{M^2} A_0(m_1, m_2, M), \quad (\text{B5})$$

$$\bar{\mathcal{F}}_2(m_1, m_2, M_1, M_2) = m_1^2 m_2^2 \left[\frac{1}{4M_W^2} A_2(m_1, m_2, M_1, M_2) + A_0(m_1, m_2, M_1, M_2) \right]. \quad (\text{B6})$$

A_0 and A_2 are given by

$$A_0(m_1, m_2, M_1, M_2) = \frac{-1}{m_1^2 - m_2^2} \left[\frac{m_1^2 \ln m_1^2}{(M_1^2 - m_1^2)(M_2^2 - m_1^2)} - \frac{m_2^2 \ln m_2^2}{(M_1^2 - m_2^2)(M_2^2 - m_2^2)} \right] \\ + \frac{-1}{M_1^2 - M_2^2} \left[\frac{M_1^2 \ln M_1^2}{(m_1^2 - M_1^2)(m_2^2 - M_1^2)} - \frac{M_2^2 \ln M_2^2}{(m_1^2 - M_2^2)(m_2^2 - M_2^2)} \right], \quad (\text{B7})$$

$$A_0(m_1, m_2, M) = \frac{-1}{m_1^2 - m_2^2} \left[\frac{m_1^2 \ln \frac{m_1^2}{M^2}}{(M^2 - m_1^2)^2} - \frac{m_2^2 \ln \frac{m_2^2}{M^2}}{(M^2 - m_2^2)^2} + \frac{1}{M^2 - m_1^2} - \frac{1}{M^2 - m_2^2} \right], \quad (\text{B8})$$

$$A_0(m, M) = \frac{-1}{(M^2 - m^2)^2} \left[\frac{(M^2 + m^2) \ln \frac{m^2}{M^2}}{M^2 - m^2} + 2 \right], \quad (\text{B9})$$

$$A_2(m_1, m_2, M_1, M_2) = \frac{1}{m_1^2 - m_2^2} \left[\frac{m_1^4 \ln m_1^2}{(M_1^2 - m_1^2)(M_2^2 - m_1^2)} - \frac{m_2^4 \ln m_2^2}{(M_1^2 - m_2^2)(M_2^2 - m_2^2)} \right] \\ + \frac{1}{M_1^2 - M_2^2} \left[\frac{M_1^4 \ln M_1^2}{(m_1^2 - M_1^2)(m_2^2 - M_1^2)} - \frac{M_2^4 \ln M_2^2}{(m_1^2 - M_2^2)(m_2^2 - M_2^2)} \right], \quad (\text{B10})$$

$$A_2(m_1, m_2, M) = \frac{1}{m_1^2 - m_2^2} \left[\frac{m_1^4 \ln \frac{m_1^2}{M^2}}{(M^2 - m_1^2)^2} - \frac{m_2^4 \ln \frac{m_2^2}{M^2}}{(M^2 - m_2^2)^2} + \frac{M^2}{M^2 - m_1^2} - \frac{M^2}{M^2 - m_2^2} \right], \quad (\text{B11})$$

$$A_2(m, M) = \frac{1}{(M^2 - m^2)^2} \left[\frac{2m^2 M^2 \ln \frac{m^2}{M^2}}{M^2 - m^2} + m^2 + M^2 \right]. \quad (\text{B12})$$

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